5. L. I. Sedov, Similarity Methods and Dimensional Analysis in Mechanics [in Russian], Nauka, Moscow (1977).
6. R. I. Nigmatulin, Foundations of the Mechanics of Heterogeneous Media [in Russian], Nauka, Moscow (1978).
7. O. V. Voinov and A. G. Petrov, "Stability of a small body in a nonuniform flow," Dokl. Akad. Nauk SSSR, 237, No. 6 (1977).
8. C. R. Carlos and J. F. Richardson, "Solids movement in liquid fluidized beds. I. Particle velocity distribution," Chem. Eng. Sci., 23, 813 (1968).
9. J. F. Davidson and D. Harrison (editors), Fluidization, Academic Press (1971).
10. A. D. Maude and R. L. Whitmore, "A generalized theory of sedimentation," Br. J. Appl. Phys., 9, No. 12 (1958).
11. L. I. Sedov, Mechanics of a Continuous Medium, Vol. 1 [in Russian], Nauka, Moscow (1976).

FORMATION OF TAYLOR VORTICES BETWEEN HEATED ROTATING CYLINDERS

V. V. Kolesov

UDC 532.516

Experimental observations [1-4] show that a secondary steady flow of the Taylor vortex type (rotationally symmetric toroidal vortex cells regularly positioned along the symmetry axis of the cylinders) can arise as the result of the loss of stability of a nonisothermal Couette flow between concentric cylinders rotating with different angular velocities. This secondary flow was found in [5] by the Lyapunov–Shmidt method in the case in which the cylinders are rotating in the same direction and the Prandtl number is equal to unity.

Results are presented in this paper of calculations of Taylor vortices both for the case in which the cylinders rotate in the same direction and for the case of an opposite rotation direction of the cylinders. The change in the structure of the vortices as the values of the parameters of the problem vary is illustrated by the pattern of the stream lines of the secondary flow. Analytic dependences of the amplitude of the vortices and the decrement of a nonisothermal Couette flow on the Prandtl number are obtained, which eliminate the need to make time-consuming calculations and permit establishing some properties of the fundamental and secondary regimes. One should note that a similar dependence of the amplitude of the secondary regime on the Prandtl number for the steady problem of free convection in a layer of liquid was established and used in the calculations in [6].

1. The Lyapunov–Shmidt Series. Let a viscous uniform heat-conducting liquid fill the cavity between two infinite solid concentric cylinders. The radii, angular velocities, and temperatures of the inner and outer cylinders will be denoted by R_1, Ω_1, Θ_1 and R_2, Ω_2, Θ_2, respectively.

We will assume that there are no external body forces and the discharge rate of the liquid through the transverse cross section of the cavity of the cylinders is equal to zero. Then the Navier–Stokes equations and the thermal conductivity equation permit an exact solution (a nonisothermal Couette flow) with the velocity vector $\mathbf{U}_0 = \{u_{0r}, u_{0\varphi}, u_{0z}\}$, temperature T_0, and pressure Π_0 (r, φ, and z are dimensionless cylindrical coordinates):

$$\mathbf{U}_0 = \{0,\ V_0(r),\ 0\},\ V_0 = ar + b/r,\ T_0 = c\ln r + 1, \tag{1.1}$$

$$\Pi_0 = \int\limits_1^r \frac{V_0^2(\rho)}{\rho}\left(1 - \frac{\mu}{\Pr}\ln\rho\right)d\rho + \text{const},$$

$$a = (\Omega R^2 - 1)/(R^2 - 1),\ b = 1 - a,\ c = (\Theta - 1)/\ln R,$$

where $\mu = \beta c \Theta_1 \Pr$ is the Rayleigh number, $\Pr = \nu/\chi$ is the Prandtl number, β, ν, and χ are the thermal expansion, kinematic viscosity, and thermal conductivity coefficients, respectively, $R = R_2/R_1$, $\Omega = \Omega_2/\Omega_1$, and $\Theta = \Theta_2/\Theta_1$.

Rostov-on-Don. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 87-93, November-December, 1981. Original article submitted May 30, 1980.

0021-8944/81/2206-0811$07.50

It is necessary to find the rotationally symmetric secondary steady flow which arises as the result of the stability loss of the flow (1.1).

Applying the procedure of [7] and using the results of [5], we are convinced that one can represent the secondary flow in the form of power series in a small neighborhood of the critical value of the Reynolds number:

$$\begin{gathered}
v_r = \varepsilon A u_1(r)\cos\alpha z + \varepsilon^2 A^2\lambda_0 u_2(r)\cos 2\alpha z + O(\varepsilon^3),\\
v_\varphi = V_0(r) + \varepsilon A v_1(r)\cos\alpha z + \varepsilon^2 A^2\lambda_0[v_2(r)\cos 2\alpha z + v_0(r)] + O(\varepsilon^3),\\
v_z = \varepsilon A w_1(r)\sin\alpha z + \varepsilon^2 A^2\lambda_0 w_2(r)\sin 2\alpha z + O(\varepsilon^3),\\
T = T_0 + c\Pr(\varepsilon A\tau_1(r)\cos\alpha z + \varepsilon^2 A^2\lambda_0[\tau_2(r)\cos 2\alpha z + \tau_0(r)]) + O(\varepsilon^3),\\
\Pi = \Pi_0 + (\varepsilon A q_1(r)\cos\alpha z + \varepsilon^2 A^2\lambda_0[q_2(r)\cos 2\alpha z + q_0(r)])/\lambda_0 + O(\varepsilon^3),
\end{gathered} \tag{1.2}$$

where α is the wave number, $\varepsilon = [(\lambda - \lambda_0)\operatorname{sign}(g)]^{1/2}$ is a small parameter, $\lambda = \Omega_1 R_1^2/\nu$ is the Reynolds number, and λ_0 is its critical value. The amplitude of the vortices A and the constant g are found from the formulas

$$\begin{gathered}
A = \sqrt{|g|}/\lambda_0, \quad g = 2I_1/I_2,\\
I_1 = \int_1^R [(2\omega_1 v_1 - \mu\omega_2\tau_1)u + (g_1 v - g_2\tau)u_1]\,r\,dr,\\
I_2 = \int_1^R (f_1 u + f_2 v + f_3 w + \Pr f_4\tau)\,r\,dr,\\
f_1 = u_1\frac{du_2}{dr} + u_2\frac{du_1}{dr} - \alpha(u_1 w_2 + 2u_2 w_1) - \frac{2}{r}v_1(v_2 + 2v_0),\\
f_2 = u_1\left(\frac{d}{dr} + \frac{1}{r}\right)(v_2 + 2v_0) + u_2\left(\frac{d}{dr} + \frac{1}{r}\right)v_1 - \alpha(v_1 w_2 + 2v_2 w_1),\\
f_3 = u_1\frac{dw_2}{dr} - u_2\frac{dw_1}{dr} - \alpha w_1 w_2,\\
f_4 = u_1\frac{d}{dr}(\tau_2 + 2\tau_0) + u_2\frac{d\tau_1}{dr} - \alpha(\tau_1 w_2 + 2\tau_2 w_1),\\
\omega_1 = V_0/r, \quad \omega_2 = V_0^2/r, \quad g_1 = -2a, \quad g_2 = 1/r.
\end{gathered} \tag{1.3}$$

In order to determine the critical value of the Reynolds number and the functions u_1, v_1, and τ_1, it is necessary to find the smallest positive eigenvalue λ_0 of the spectral problem

$$\begin{gathered}
L_\alpha^2 u_1 = \alpha^2\lambda(2\omega_1 v_1 - \mu\omega_2\tau_1), \quad L_\alpha v_1 = -\lambda g_1 u_1,\\
\left(L_\alpha + \frac{1}{r^2}\right)\tau_1 = \lambda g_2 u_1, \quad L_\alpha \equiv \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2} - \alpha^2,\\
du_1/dr = u_1 = v_1 = \tau_1 = 0 \ (r = 1,\ R).
\end{gathered} \tag{1.4}$$

The functions w_1 and q_1 are of the form

$$w_1 = -\frac{1}{\alpha r}\frac{d}{dr}(ru_1), \quad q_1 = -\frac{1}{\alpha}\left(L_\alpha + \frac{1}{r^2}\right)w_1.$$

It is necessary to solve the inhomogeneous boundary-value problem

$$\begin{gathered}
L_{2\alpha}^2 u_2 = 4\alpha^2\lambda_0(2\omega_1 v_2 - \mu\omega_2\tau_2) + \psi_1,\\
L_{2\alpha}v_2 = -\lambda_0 g_1 u_2 + \psi_2, \ (L_{2\alpha} + 1/r^2)\tau_2 = \lambda_0 g_2 u_2 + \Pr\psi_3,\\
du_2/dr = u_2 = v_2 = \tau_2 = 0 \ (r = 1,\ R),\\
\psi_1 = \frac{2\alpha^2}{r}v_1^2 - \left(\frac{du_1}{dr} + \frac{2}{r}u_1 - u_1\frac{d}{dr}\right)L_\alpha u_1,\\
\psi_2 = \frac{1}{2}\left(u_1\frac{dv_1}{dr} - v_1\frac{du_1}{dr}\right), \quad \psi_3 = \frac{1}{2}\left(u_1\frac{d\tau_1}{dr} + \alpha\tau_1 w_1\right)
\end{gathered} \tag{1.5}$$

in order to determine the functions u_2, v_2, and τ_2.

The functions w_2 and q_2 are of the form

$$w_2 = -\frac{1}{2\alpha r}\frac{d}{dr}(ru_2), \quad q_2 = \frac{1}{4\alpha}\left(u_1\frac{dw_1}{dr} + \alpha w_1^2\right) - \frac{1}{2\alpha}\left(L_{2\alpha} + \frac{1}{r^2}\right)w_2.$$

It is necessary to solve the homogeneous boundary-value problem

$$L_\alpha^2 u = \alpha^2\lambda_0(g_1 v - g_2\tau),$$
$$L_\alpha v = -2\lambda_0\omega_1 u, \quad (L_\alpha + 1/r^2)\tau = \lambda_0\mu\omega_2 u,$$
$$du/dr = u = v = \tau = 0 \quad (r = 1,\ R) \tag{1.6}$$

in order to determine the functions u, v, and τ.

The functions w, q_0, v_0, and τ_0 are of the form

$$w = -\frac{1}{\alpha r}\frac{d}{dr}(ru), \quad q_0 = \lambda_0\int_1^r (2\omega_1 v_0 - \mu\omega_2\tau_0)\,d\rho - \varphi_1,$$
$$v_0 = r\varphi_2(r) - \varphi_2(R)R^2(r^2-1)/(R^2-1)r, \quad \tau_0 = \varphi_3(r) - \varphi_3(R)\ln r/\ln R,$$
$$\varphi_1 = \frac{u_1^2}{2} + \frac{1}{2}\int_1^r \frac{u_1^2 - v_1^2}{\rho}\,d\rho, \quad \varphi_2 = \frac{1}{2}\int_1^r \frac{u_1 v_1}{\rho}\,d\rho, \quad \varphi_3 = \frac{\Pr}{2}\int_1^r u_1\tau_1\,d\rho.$$

We will apply the perturbation method [8] to investigate the arrangement of the stability spectra of the flows (1.1) and (1.2).

Superimposing infinitely small rotationally symmetric monotonic $(2\pi/\alpha)$-periodic perturbations proportional to $\exp(\sigma t)$ on the flow (1.1), linearizing the boundary-value problem obtained in the neighborhood of the flow (1.1), and decomposing the eigenvalue σ, which disappears as $\lambda\to\lambda_0$, into a series of perturbation theory, we obtain

$$\sigma = \sigma_2(\lambda - \lambda_0) + O[(\lambda-\lambda_0)^2], \quad \sigma_2 = I_1/\lambda_0 I_3,$$
$$I_3 = \int_1^R (u_1 u + v_1 v + w_1 w + \Pr\tau_1\tau)\,r\,dr. \tag{1.7}$$

Similarly, the eigenvalue σ' from the stability spectrum of the flow (1.2), which disappears as $\lambda\to\lambda_0$, is decomposed into the series

$$\sigma' = \sigma_2'\varepsilon^2 + O(\varepsilon^4). \tag{1.8}$$

The decrements σ_2 and σ_2' of the flows (1.1) and (1.2) are connected by the relationship

$$\sigma_2' = -2\sigma_2\,\mathrm{sign}(g). \tag{1.9}$$

We will assume that the "first" eigenvalue λ_0 of the problem (1.4) is simple and the constants I_1, I_2, and I_3 are different from zero.

Let $\sigma_2 > 0$; then the nonisothermal Couette flow (1.1) is stable in the case of small subcriticalities ($\lambda < \lambda_0$) with respect to rotationally symmetric monotonic $(2\pi/\alpha)$-periodic perturbations, and it is unstable in the case of small supercriticalities ($\lambda > \lambda_0$). If $g > 0$, then when the Reynolds number λ passes through the critical value λ_0 a rotationally symmetric $(2\pi/\alpha)$-periodic secondary steady flow (1.2) which is stable for small supercriticalities with respect to the monotonic perturbations of the same symmetry and periodicity bifurcates from the flow (1.1). If $g < 0$, then the secondary flow (1.2) branches off into the subcritical region and is unstable for small subcriticalities.

Using the results of [7], one can convince oneself that the secondary flow (1.2) is determined in a unique way (to the accuracy of a shift along the cylinder axis z) by the wavenumber α with fixed μ, Pr, Ω, and R.

2. Dependence of the Parameters of the Secondary Flow on the Prandtl Number. The solutions u_1, v_1, and τ_1 and u, v, and τ of the homogeneous problems (1.4) and (1.6) do not depend upon Pr. The solution u_2, v_2, and τ_2 of the inhomogeneous problem (1.5) permits the representation

$$u_2 = u_2^{(1)} + \Pr u_2^{(2)}, \quad v_2 = v_2^{(1)} + \Pr v_2^{(2)},$$
$$\tau_2 = \tau_2^{(1)} + \Pr\tau_2^{(2)},$$

where $u_2^{(k)}$, $v_2^{(k)}$, $\tau_2^{(k)}$ (k = 1, 2) do not depend upon Pr. We obtain from (1.3), (1.7), and (1.8)

$$A = \sqrt{|g|/\lambda_0},\quad g = a_0 + a_1\mathrm{Pr} + a_2\mathrm{Pr}^2, \qquad (2.1)$$
$$\sigma_2' = -2\sigma_2\,\mathrm{sign}\,(g),\quad \sigma_2 = 1/(b_0 + b_1\mathrm{Pr}),$$

where the constants a_0, a_1, a_2, b_0, b_1 depend only on the wavenumber α, the Rayleigh number μ, the ratio of the angular velocities of the cylinders Ω, and the ratio of the cylinder radii R, and they do not depend upon Pr.

3. Numerical Results. The spectral problem (1.4), the inhomogeneous boundary-value problem (1.5), and the homogeneous problem (1.6) converged to boundary-value problems for eight ordinary first-order differential equations with variable coefficients, each of which was solved numerically by the ranging method. An orthogonalization procedure was applied to suppress the rapidly increasing solutions which arise at large λ. The characteristic solution of the problem (1.4) was specified with the help of the normalization condition

$$\int_1^R u_1(r)\,r\,dr = 1.$$

From the physical standpoint the most dangerous perturbations are of greatest interest, therefore numerical minimization of λ_0 with respect to the wave number α was performed:

$$\lambda_* = \min_\alpha \lambda_0(\alpha) = \lambda_0(\alpha_*).$$

One should note here that for some values of the problem parameters, oscillatory perturbations prove to be more dangerous than monotonic ones [9, 10]. As a result of the loss of stability of the flow (1.1), nonsteady Taylor vortices discussed in this paper and two-dimensional or three-dimensional self-oscillations arise first.

The calculations were performed for the case R = 2. The results are presented in Table 1 and Figs. 1-4. The calculations have shown that in contrast to the isothermal case (μ = 0) the decrement σ_2 of the regime (1.1) can turn out to be negative. This indicates that in a certain bilateral neighborhood of the point $\lambda = \lambda_0$ the nonisothermal Couette flow (1.1) is unstable. One can verify that when $\lambda = 0$ all the eigenvalues σ from the stability spectrum Σ of the flow (1.1) are negative. It follows from (1.7) that when $\lambda < \lambda_0$ a positive σ occurs in the spectrum Σ. It follows from this that for some λ a merging of the eigenvalues σ from the spectrum Σ has occurred with the formation of a complex-conjugate pair, which for $\lambda = \lambda^{(0)}$ $(0 < \lambda^{(0)} < \lambda_0)$ has passed through the imaginary axis into the right half-plane ($\sigma^{\mathrm{Re}} \geqslant 0$), i.e., an oscillatory loss of stability of the flow (1.1) has occurred (a calculation of the neutral curves of oscillatory rotationally symmetric instability of a nonisothermal Couette flow was made in [9] in the case of an infinitely small gap between the cylinders). The calculations show that upon a further increase in the Reynolds number λ this complex-conjugate pair merges into a double real eigenvalue σ, which then splits into two simple ones. One of them returns along the real axis to the origin of coordinates and

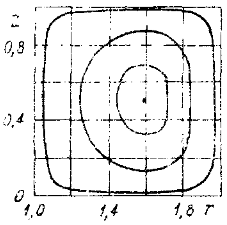

Fig. 1

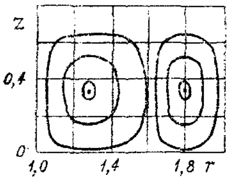

Fig. 2

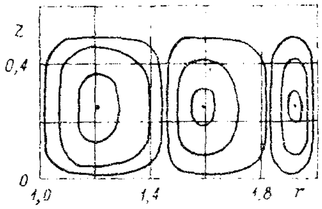

Fig. 3

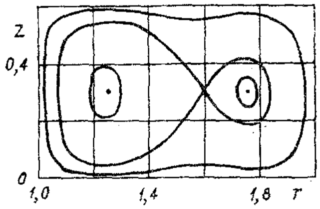

Fig. 4

TABLE 1

μ	Ω	α_*	λ_*	a_0	a_1	a_2	b_0	b_1
10	1,0	3.09	16,8	—0,040	—0,006	0,165	—0,85	11,87
10	0,4	3.12	24,7	—0,014	0,004	0,174	5,47	17,86
10	0.0	3,20	35,7	0,076	0,025	0,164	24,16	22,29
10	—0.2	3,30	45,5	0,206	0,070	0,167	43,71	26,91
10	—0,4	3,55	60,7	0.358	0,126	0,245	54,78	54,54
10	—0,5	3,78	71,1	0.335	0.040	0,348	39,46	93,36
10	—0.6	4.11	83,6	0,197	— 0,181	0,481	7,38	147,93
10	—0,7	4,59	98,0	—0,019	—0,320	0,570	—17.94	189,31
10	—0,8	5,27	114.3	—0,198	—0,146	0,543	—1,98	177,38
4	0,8	2,98	59,8	—2,396	—1,152	2,728	—438,4	592,4
4	0,6	3,08	48,3	—0,422	- 0,081	0,837	—73,0	165,6
4	0,4	3.12	44,6	—0,102	0,001	0,414	0,3	76,1
4	0.2	3.15	44,4	0,040	0,016	0,239	31,3	43,0
4	0,0	3.19	47.6	0,177	0,030	0,152	54,8	28,2
4	—0.2	3.29	56.3	0,406	0,071	0,126	83,7	25,4
4	—0,4	3,86	77,8	0,621	0,125	0,232	106.3	43,7
4	—0,6	5,05	109,1	0.402	0,133	0,508	123,1	47,2
4	—0.8	6.12	142.6	—0,193	0,180	0,748	151,5	46,2
—4	0,0	3,67	215,2	17.72	—11,34	—2,33	4770	—3501
—4	—0,1	3.14	113,8	4.54	—2,05	—0,91	1064	—578
—4	—0,2	3.23	103,4	3.46	—1,33	—0,68	666	—288
—4	—0,3	3,75	117,0	3.57	—1,26	—1,06	613	—262
—4	—0,4	4,50	134,2	3,43	—1,02	—1,51	516	—198
—4	—0,5	5,05	149,9	3.31	—1,01	—1,80	479	—163
—4	—0,6	5.58	166,7	2,78	—0,97	—1,94	464	—143

vanishes for $\lambda = \lambda_0$. It is precisely this eigenvalue which is expandable into the series (1.7). The other eigenvalue remains positive for $\lambda = \lambda_0$, which indicates instability of the flow (1.1) in some neighborhood of the point $\lambda = \lambda_0$. The secondary steady flow (1.2) generated for $\lambda = \lambda_0$ is also unstable when $\sigma_2 < 0$, since only an unstable regime can bifurcate from an unstable regime.

A change in the sign of the decrement σ_2 upon a variation of the problem parameters (μ, Pr, Ω, and R) occurs as a result of the change in sign of the constant I_3. The neutral curves of oscillatory and monotonic instability merge when $I_3 = 0$. The series of perturbation theory for an eigenvalue $\sigma \in \Sigma$ which disappears as $\lambda \to \lambda_0$ is of the form

$$\sigma = \pm \sigma_1 \sqrt{\lambda - \lambda_0} + O(\lambda - \lambda_0)$$

in the case $I_3 = 0$. The latter indicates that the flow (1.1) is unstable for $\lambda > \lambda_0$ in the case $I_3 = 0$.

We note that the discussions given above are barely connected with the characteristics of the nonisothermal Couette flow and can be used for the detection of the oscillatory stability loss of other steady flows.

We will point out one more difference from the isothermal case. When $\mu \neq 0$ the branching can be directed into the subcritical region for the most dangerous perturbations; therefore not only mild but also rigorous onset of nonisothermal Taylor vortices is possible in experiments.

Pictures of the stream lines of nonisothermal Taylor vortices calculated according to the linear problem are shown in Figs. 1-4.

For large values of the Rayleigh number ($\mu > 0$) a single vortex cell occurs on the meridian plane in the region $D(\{1 \leqslant r \leqslant R, 0 \leqslant z \leqslant \pi/\alpha_*\})$ whose symmetry center is located near the middle of the segment $1 \leqslant r \leqslant R$ (Fig. 1, $\mu = 40$, $\Omega = -0.7$, $\alpha_* = 3.135$, and $\lambda_* = 22.53$). Variation of the ratio of the angular velocities of the cylinders Ω results only in an insignificant deformation of the cell.

For small $\mu > 0$ a single vortex cell also occurs in the region D if the cylinders rotate in the same direction ($\Omega \geqslant 0$) or in different directions ($\Omega < 0$) but the absolute magnitude of Ω is small. If $\Omega \ll 0$, then as Ω decreases, a second (Fig. 2, $\mu = 2$, $\Omega = -0.5$, $\alpha_* = 4.655$, and $\lambda_* = 102.75$) and then a third (Fig. 3, $\mu = 2$, $\Omega = -0.8$, $\alpha_* = 6.277$, and $\lambda_* = 153.81$) vortex cell arise near the surface of the outer cylinder.

If $\mu < 0$, then several vortex cells can arise both for $\Omega < 0$ and for $\Omega > 0$. A second vortex cell arises near the outer cylinder if $\Omega < 0$ and near the inner cylinder if $\Omega > 0$.

For some values of the parameters of the problem more complicated vortex structures can exist. For example, when $\mu = 10$ and $\Omega = -0.8$ (Fig. 4, $\alpha_* = 5.271$ and $\lambda_* = 114.24$) a large vortex occurs in the region D inside of which are located two small vortices. The rotation of the liquid in all three vortices occurs in the same direction.

The author is grateful to V. I. Yudovich for his constant attention to this research.

LITERATURE CITED

1. I. S. Bjorkland and W. H. Kays, "Heat transfer between concentric rotating cylinders," Trans. Am. Soc. Mech. Eng., Ser. C, Heat Transfer, 81, 175 (1959).
2. F. Tachibana, S. Fukui, and H. Mitsumura, "Heat transfer in an annulus with an inner rotating cylinder," Bull. Jpn. Soc. Mech. Eng., 3, No. 9 (1960).
3. F. C. Haas and A. H. Nissan, "Experimental heat transfer characteristics of a liquid in Couette motion and with Taylor vortices," Proc. R. Soc. A, 261, 215 (1961).
4. Becker and Kay, "An investigation of a nonadiabatic flow in an annular channel with an inner rotating cylinder," Trans. Am. Soc. Mech. Eng., Ser. C, Heat Transfer, 84, No. 2 (1962).
5. V. V. Kolesov and S. N. Ovchinnikova, "Calculation of the secondary steady flow between rotating cylinders with a radial temperature gradient," Izv. Severo-Kavkazsk. Nauchn. Tsentr. Vyssh. Shkol., Ser. Estestven. Nauki, No. 4 (1975).
6. É. É. Karamanova and G. K. Ter-Grigor'yants, "Calculation of plane steady convection," Izv. Severo-Kavkazsk. Nauchn. Tsentr. Vyssh. Shkol., Ser. Estestven. Nauki, No. 3 (1977).
7. V. I. Yudovich, "Free convection and branching," Prikl. Mat. Mekh., 31, No. 1 (1967).
8. V. I. Yudovich, "Stability of convection flows," Prikl. Mat. Mekh., 31, No. 2 (1967).
9. Lee, "The effect of thermal diffusion on the stability of the flow between two rotating cylinders," Trans. Am. Soc. Mech. Eng., Ser. F, Lubricant Technology, 99, No. 3 (1977).
10. V. V. Kolesov, "The stability of a nonisotherml Couette flow," Izv. Akad. Nauk SSSR, Mekh. Zhid. Gaza, No. 1 (1980).